Systematic Errors in Micrometer Measurements using Different Methods for Finding Polar Cap Latitudes of Mars

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INTRODUCTION

For many years the ALPO Mars Section has engaged in micrometer measurements of Mars to plot the retreat and reformation of the planet's polar caps. The small apparent size of Mars renders micrometer measurement very difficult and laborious task amateur astronomers are not likely to enjoy. You might compare this type of observing to having a tooth filled without the benefit of Novocain. However, it is an important method for studying the behavior of Mars' polar caps and their effect on the planet's weather. Adding to this difficulty is the variable "astronomical seeing" conditions that causes the apparent disk of Mars to blur or expand. Clouds and hazes at the Martian polar regions also hamper measurements.

During the 1960's and 1970's experiments in measuring Martian polar caps from photographs were carried out by the ALPO Mars Section. A two degree (2°) systematic error in latitude was found by the late Charles F. ("Chick") Capen using this method even on photographs taken with a variety of telescopes and apertures. Measurements made by camera were compared with those made with bi-filar micrometer using the same telescope and plate scale. Photographs taken during less than ideal "astronomical seeing" resulted in increased errors. No empirical method has been found to correct for these systematic errors. So, we adopted the bi-filar micrometer, and in some cases the reticule micrometer, as our main polar cap measuring device.

In recent years we have attempted to employ an alternate method of measuring the Martian polar caps used by other observing organizations. We have found this method to be outmoded and to produce high rates of systematic errors. We also feel that measuring the Martian polar cap from drawings produces errors in excess of those by the photographs or micrometer. Also, we use a deep red filter in the optical system while measuring the Martian polar caps to penetrate through the planet's atmosphere. A red filter also reduces the effects of poor seeing and irradiation of the bright planet in the micrometer eyepiece.

MEASURE THE DISK OF MARS

The current ALPO Mars Recorders continued to use an old and proven Direct -Indirect Method of measuring the polar caps with a bi-filar micrometer [***Peek*,** 1981]. This method is desirable because it reduces possible mechanical errors in the bi-filar micrometer and eliminates the need to compute the micrometer zero. In both methods discussed below the disk of Mars is measured using the Direct-Indirect method discussed next (See Figure 1).

To measure with Direct-Indirect method an object: first indirectly (**In**) measured between the fixed and movable webs, then, the object is positioned across the fixed web to the other side. The movable web (**M**) is adjusted across the fixed web (**F**) and the object is then measured directly (**Di**) with the opposite screw adjustments [***Beish et al*,** 1986]. The results are averaged and the separation of the webs are determined by:

Separation (**D**) = (Di - In) / 2 – Wt

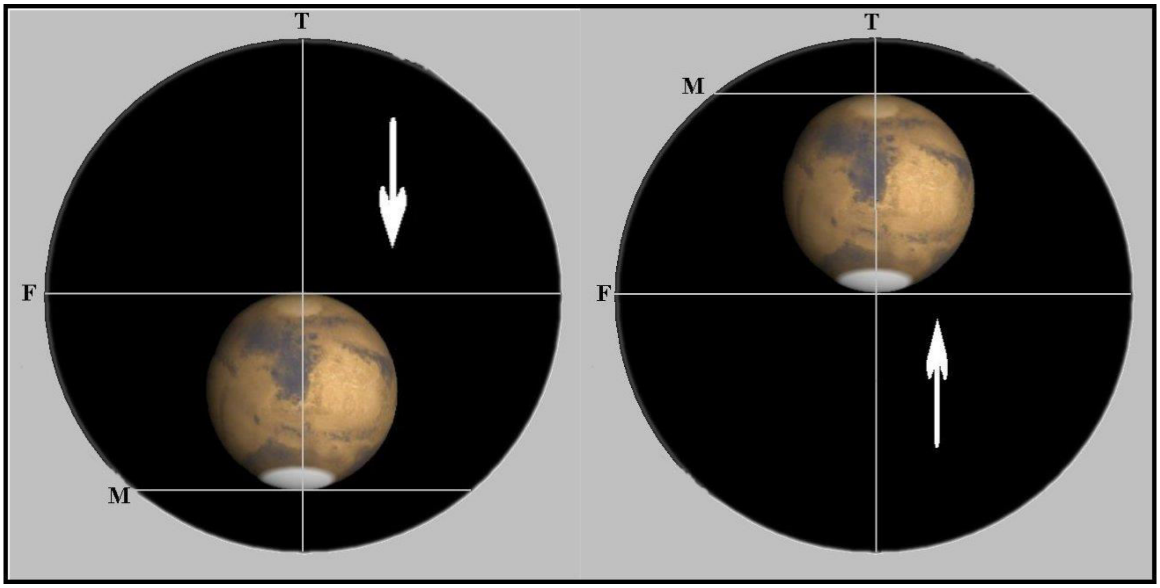

Figure 1. Appearance of Mars in micrometer eyepiece using ALPO method. In the *left* image, Mars is situated between web (M) and web (F) in Direct side. *Right* image Mars is moved down by the telescope drive and situated between web (F) and web (M) in the Indirect side. The movable web (M) is adjusted down and across the fixed web (F). The web (T) is used for centering. Measuring the apparent disk (D) of Mars will be used in both methods described below.

Now, measuring the apparent disk diameter we find Direct (**Di**) = 11.10757 mm and Indirect (**In**) = 10.24683, therefore the separation (**D**) of the webs. In this case the micrometer web (**Wt**) is 0.012 mm thick:

$$\mathbf{D} = (11.10757 - 10.24683) / 2 - 0.012 = 0.86074 / 2 - 0.012 = 0.41837$$

ALPO METHOD

Second, the polar cap breadth, or East to West span across the sphere of the planet is measured. Then the apparent disk of Mars is measured from north to south to be used in a standard spherical geometric equation:

$$\text{Latitude} = \arccos (C / D),$$

where **C** is the breadth of the cap and **D** is the apparent diameter of the disk.

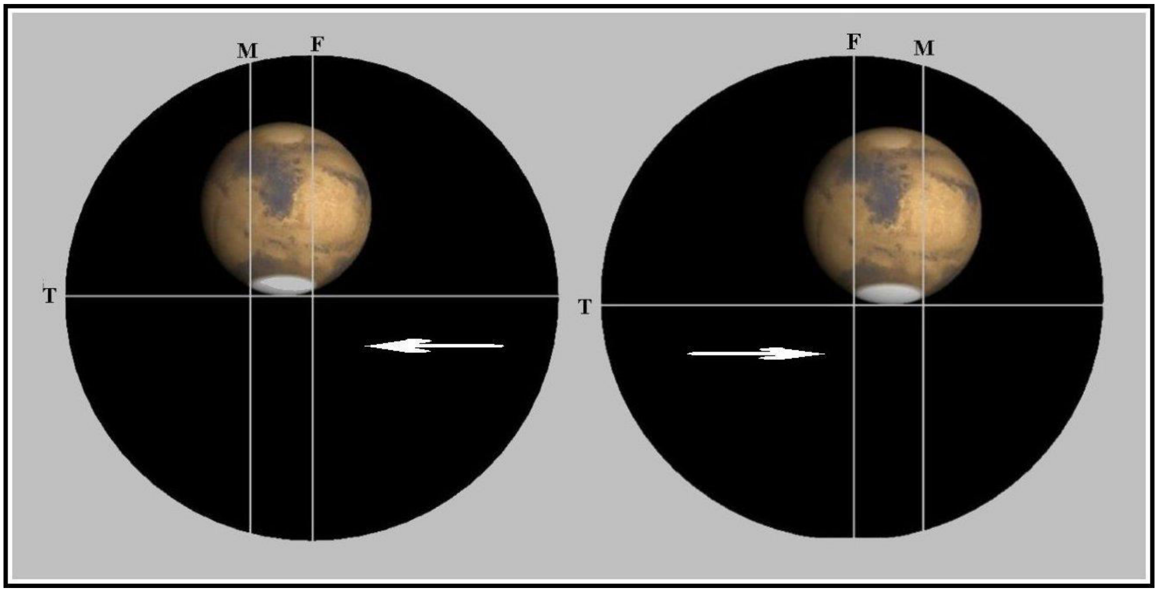

Figure 2. Appearance of Mars in micrometer eyepiece using ALPO method. In the *left* image, the apparent polar cap (C) is situated between web (M) and web (F) in Direct side. *Right* image Mars is moved left by the telescope drive and the apparent polar cap (C) is situated between web (F) and web (M) in the Indirect side. The movable web (M) is adjusted left and across the fixed web (F). The web (T) is used for centering.

This results in straightforward latitude of the edge of each side of the polar cap, or the co-latitude can be found by the equation: $\mathbf{2 \sin^{-1} (1 - C/D)}$. [***Beish, et al,*** 1986]. This method and calculation has proven quite good and produces less than 0.5 degree systematic errors (see Figure 1 and Figure 2).

For example, let's consider a typical telescope and micrometer used by ALPO's Mars Recorders during the last apparition. We might have measured the north polar cap of Mars when the planet was 10 seconds of arc apparent diameter and a declination of the Earth ($\mathbf{D_e}$) of 10 degrees. Our observing period would be under perfect seeing conditions, and then under less than perfect conditions where the planet would expand by one-micrometer web or 0.012 mm. Using the above micrometer screw constant of 9.526 arcsec/mm the expansion of 0.012mm would correspond to slightly more than 1/8th of an arcsec during less than perfect seeing. Not a very large difference in seeing or image blurring.

Now, we will use the standard ALPO micrometer method, measuring Mars' polar cap width (east-west) resulting in a cap width of 0.21253 mm and the apparent disk diameter as 0.41837mm and for the alternate method a cap thickness (north-south) of 0.04662mm: where the cap width (**C**) = 0.21253 mm and disk (**D**) = 0.41837 mm

$$\text{Latitude} = \cos^{-1} (C / D) = \cos^{-1} (0.507995) = 59.5°$$

Now, let's add the 0.012mm web image expansion or blur due to less than perfect seeing conditions and we have a cap width of 0.22453 mm or the disk of 0.43037 mm respectively; then find:

$$\text{Latitude} = \cos^{-1} (0.22453 / 0.43037) = 58.6°, \text{error} = -0.9°$$

A systematic error would yield - 0.9° in the ***ALPO*** method.

ALTERNATE METHOD

We will begin by discussing the ***alternate method*** of measuring the polar latitudes of Mars [***Dollfus,*** 1973], [***Minami,*** 1993], [***Minami,*** 1993]. The alternate method often uses drawings to determine the polar cap latitudes of Mars and may be subject to additional systematic errors due to personal errors or hand-eye coordination problems.

First, find the planetocentric angle (**β**) of the polar cap by measuring the north-south thickness of the polar cap as shown in figure 3. The apparent disk (0.41837 mm) is measured as shown in figure 1 and the radius is determined by dividing this diameter by 2. Then take the absolute value the Declination of Earth (**De**) to find the planetocentric latitude of the Earth (| ϕ |) and apply these to the following geometric equation:

$$\beta = \cos\text{-}^{-1} (1 - d / r) - | \phi |,$$

where **β** is the planetocentric angle of the polar cap, **De** is the Declination of Earth, **d** is the thickness of the cap, and **r** is the radius of the apparent disk (See Figure 3).

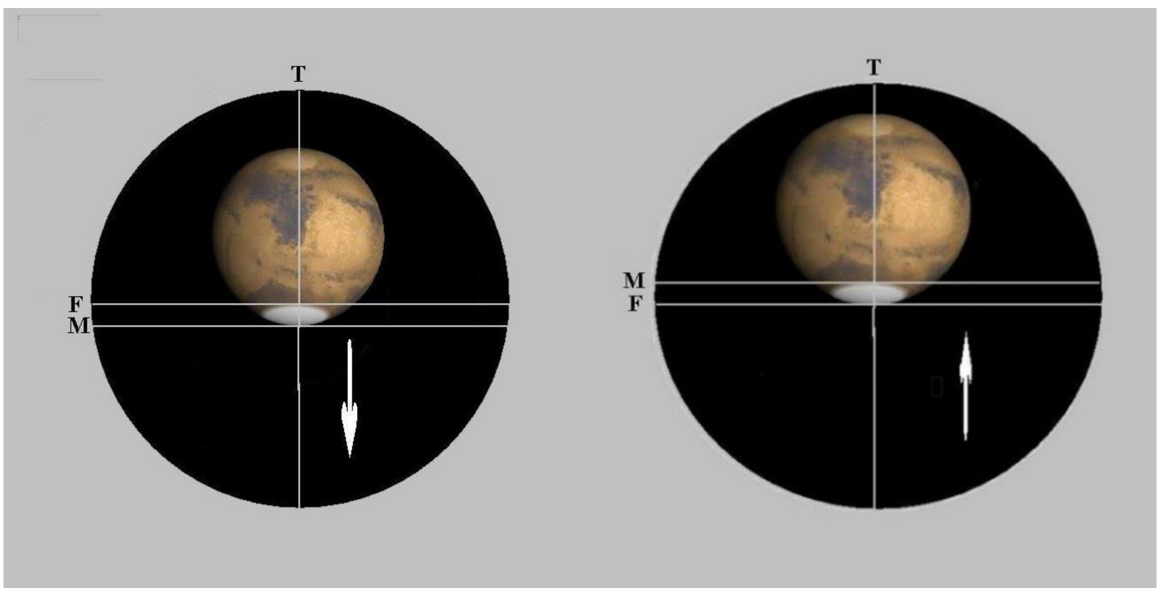

Figure 3. Appearance of Mars in micrometer eyepiece using ALTERNATE method. In the *left* image, the apparent polar cap (C) is situated between web (M) and web (F) in Direct side. *Right* image Mars is moved down by the telescope drive and the apparent polar cap (C) is situated between web (F) and web (M) in the Indirect side. The movable web (M) is adjusted down and across the fixed web (F). The web (T) is used for centering.

For the ALTERNATE method with a D_e of 10°and cap thickness (**d**) = 0.05012 mm, radius (**r**) of the disk of Mars = 0.41837 / 2 or 0.209185 mm

$$\beta = \cos^{-1} (1 - d / r) - | \phi | = \cos^{-1} (1 - 0.05012 / 0.209185) - 10°$$
$$= \cos^{-1} (0.7604) = 40.5° - 10° = 30.5°$$

NOTE: The latitude (**L**) of the polar cap at the central meridian = 90 – **β**, so 90° – 30.5°= 59.5°

However, even using a high quality bi-filar micrometer a slight misalignment or position error, due to blurring of the image in the webs while measuring the thickness of the cap can produce large errors in latitude. Another problem arises when the $\underline{D_e}$ approaches zero or becomes less than the desired tilt angle of the polar cap of interest. Also, the phase defect or terminator can cover much of the backside of the polar cap near some oppositions, therefore, causing a large observational and calculation error!

Using the ***ALTERNATE*** method with 0.012 mm blurring: cap thickness (**d**) = 0.06212 mm [0.05012 + 0.012] and a radius (r) 0.20919 mm to 0.22119 mm, we calculate:

$$\beta = \cos^{-1} (1 - 0.06212 / 0.215185) = 44.7° - 10° = 34.7°,$$

Measuring the polar cap in less than perfect seeing conditions that would expand and contract Mars's disk by 0.012mm or one web thickness, a systematic error would yield +4.2° would result in using the ***ALTERNATE*** method (proof: 34.7° - 30.5° = 4.2°).

Given the systematic error of 0.9° in the ***ALPO*** method, the difference between the two methods is 4.2° – 0.9° = 3.3° This error does not fit well with the two degree (2°) systematic error found by Capen using photographs [***Capen***, 1970]. It is never the less a significant error and as can be readily seen the errors using the Alternate method can be as much as 3.7 times higher than with the ALPO method. When seeing blurs the image worse than one web, say two or three webs -- then error will be even higher. Observers using micrometers are encouraged to use both methods to determine for themselves what is best for their equipment and conditions.

There is one problem -- the Martian phase. The Red Planet can have a significant phase defect near quadrature when Earth-based observers will see less than 85% of its disk illuminated. However, if the angle of the phase defect (terminator) is in the opposite hemisphere from the De, i.e., the De is positive and the PA of defect is negative, then the terminator cusp will be clear of the edge of the polar cap. In this case there will be no problems with our calculations. See ***Appendix A*** for equations for handling the effects of the phase defect on the polar cap.

COMPARE ALTERNATE METHOD WITH ALPO METHOD

These errors do not fit well with the two degree (2°) systematic error found by Capen using photographs [***Capen,*** 1970]. It can be readily seen that the errors using the Alternate method can be many times higher than with the ALPO method. When seeing blurs the image worse than one web, say two or three webs, then error will be even higher. Observers using micrometers are encouraged to use both methods to determine for themselves what is best for their equipment and conditions.

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